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# **FILTERING BOUNDARY CONDITIONS FOR LES AND EMBEDDED BOUNDARY SIMULATIONS**

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## **1. Introduction**

In the Large Eddy Simulation (LES) of turbulence, we simulate the spatially filtered velocity field. The governing equations are obtained by applying a spatial filter to the Navier-Stokes equations. The set of filtered equations are solved numerically to obtain the solution in terms of filtered quantities. Though successful LES have been performed for several different flows (Lesieur and Métais, 1996; Meneveau and Katz, 2000), LES for wall bounded flows poses some critical challenges (Piomelli et al., 1989; Balaras et al., 1996). Some of the difficulties in LES modeling of wall bounded flows are due to the strong inhomogeneity of the turbulence, the scaling of the largest scales near the wall, and the strong filter inhomogeneity commonly employed to accommodate the boundary conditions. Because of these issues, many of the reported LES of wall bounded flows have no filtering at all in the wall normal direction. The problem associated with strong filter inhomogeneity can be thought of as arising from the inconsistency of the requirement to represent a sharp boundary in a simulation which resolves only large scales.

In a different context, immersed boundary methods are promising tools for treating complex geometries in fluid dynamics simulations. In these methods the irregular boundary is embedded in a regular grid and a forcing term is included to account for the embedded boundary. This forcing is such that a prescribed velocity is achieved at the given surface boundary. The forcing is nonzero only in the neighborhood of the boundary (few grid points near the wall), but due to incompressibility of the flow, the forcing affects the entire flow field. The forcing can be considered to be singular field in the computational domain. The concept was implemented by Peskin

(1972) to simulate flow around heart valves, and Goldstein et. al. (1993) used this technique to simulate flows in complex geometries using a pseudo spectral method. Due to the global nature of the expansion functions used in the spectral methods, discontinuities produce Gibbs phenomenon in the flow field. According to Goldstein et. al. (1993), this oscillation does not corrupt the flow field as the flow evolves in time. Mohd-Yusof (1997,1998) used a different expression for the forcing to remove the severe time step restriction of the method developed by Goldstein et. al. (1993). This approach was used by Verzicco et. al. (1998) to perform an LES in complex geometries. However, in immersed boundary techniques it is in general difficult to preserve higher order accuracy and high resolution as is generally required in turbulent simulations. This difficulty mainly arises due to the presence of discontinuities at the boundaries.

These two related observations lead us to propose a new approach for embedding boundaries in the context of LES. In this approach, we treat the wall by 'filtering through it'. A homogeneous or nearly homogeneous filter is applied to the flow field including the embedded boundary. As a result, the boundary is no longer a sharp interface, but is diffused across the filter width. To recover a Direct Numerical Simulation (DNS), the filter width is made small enough to resolve all relevant scales of motion, but remains finite so that the boundary is not a sharp interface. In LES, the scale on which the boundary is resolved is consistent with the resolved scales of motion.

## 2. Governing equations and numerical methods

To illustrate the concept of 'filtering through the wall', we consider the heat equation on  $y \in [-1, 1]$ . In the computational domain, we include a buffer region outside the boundaries, where the velocities are zero.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} \quad |y| \leq 1 \quad (1)$$

$$u = 0 \quad |y| > 1 \quad (2)$$

These two equations can be combined by introducing a Heaviside function  $H(y)$ .

$$\frac{\partial u}{\partial t} = H(y) \frac{\partial^2 u}{\partial y^2} \quad -L \leq y \leq L$$

where

$$H(y) = \begin{cases} 1 & : |y| \leq 1 \\ 0 & : \text{otherwise} \end{cases}$$

A homogeneous filter is applied on the domain  $-L$  to  $L$ , including the boundary. For any function  $f$ , the filtered function  $\tilde{f}$  is obtained by

$$\tilde{f} = \int_{-L}^L G(x - \xi) f(\xi) d\xi \quad (3)$$

where  $G$  is the filter kernel. The resulting filtered equation is

$$\frac{\partial \tilde{u}}{\partial t} = \frac{\partial^2 \tilde{u}}{\partial y^2} + G(y-1) \frac{\partial u}{\partial y}(1) - G(y+1) \frac{\partial u}{\partial y}(-1)$$

$$-L \leq y \leq L$$

The boundary term,  $b$  is defined:

$$b = G(y-1) \frac{\partial u}{\partial y}(1) - G(y+1) \frac{\partial u}{\partial y}(-1)$$

which includes the unfiltered derivatives at the boundary. These are not known from the simulated filtered field. We propose to estimate the stress by applying a suitable constraint on the field in the buffer domain,  $|y| > 1$ . The boundary terms obtained by this technique are similar to the forcing term used by (Goldstein et al., 1993) except the forcing is distributed in the domain according to the filter kernel.

Now consider the Navier-Stokes equations for incompressible flows and apply a similar technique. After filtering the equations and taking account of the filtered boundary, the filtered equations are

$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0$$

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = - \frac{\partial \tilde{p}}{\partial x_i} + \frac{1}{Re} \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_j} \tilde{u}_i + b_i$$

where  $b_i$  is the boundary term. This boundary term is expressed as

$$b_i(\mathbf{x}) = \int_{\partial R} \sigma_{ij}(\mathbf{x}') n_j G(\mathbf{x} - \mathbf{x}') d\mathbf{x}'$$

where  $\sigma$  is the stress at the boundary, including pressure and viscous stress,  $\partial R$  is the boundary of the fluid region  $R$  and  $n_j$  is the unit normal to the surface.

In simulating wall bounded turbulent flow, a large number of grid points are required to resolve the near wall layer, as a result, in many practical applications it would be difficult to apply an LES that resolves the wall layer. In many LES of wall bounded flows, approximate boundary conditions are used to model the effect of the wall layer (Balaras et al., 1996).

The approximate boundary conditions are prescribed in terms of the wall shear stress, so wall stresses must be determined in terms of the resolved velocities.

In the present formulation, the unfiltered wall stresses are also required, and for analogous reason. Even though the boundary, or the flow near it is not resolved in LES, it is necessary to represent the transfer of momentum from the flow to the wall, which is governed by the wall stresses. In the current description, in which the unfiltered velocity is zero in the buffer domain, the wall stress is the surface forcing required to ensure that momentum is not transferred to the buffer domain. That is, that the velocity remains zero. This suggests a technique for determining the wall stress. Instead of defining a force to make the velocity zero at the boundary as in Mohd-Yusof (1997), we choose  $\sigma_{wall}$  to minimize the transport of momentum to the exterior domain. To this end, the wall stresses at each time step is defined by minimizing

$$E = \int_B |\tilde{\mathbf{u}}|^2 + \alpha \left| \frac{\partial \tilde{\mathbf{u}}}{\partial t} \right|^2 dx \quad (4)$$

where the integral is over the buffer domain. The  $|\tilde{\mathbf{u}}|^2$  term forces the energy in the buffer domain to be small, and the  $\alpha \left| \frac{\partial \tilde{\mathbf{u}}}{\partial t} \right|^2$  term ensures that the transfer of energy into the domain is small. The constant  $\alpha$  controls the balance between these two competing requirements and is set to a value of order  $\Delta t^2$ . In the Fourier spectral method employed in section 3 for the channel flow, this minimization is straight forward since it can be done independently for each  $(k_x, k_z)$  wavenumber, resulting in a 6-parameter optimization in  $(\sigma_{xy}, \sigma_{yy}, \sigma_{zy})$ .

### 3. Results and Discussion

In this section, we present the results of two numerical tests of the filtered boundary technique.

In both cases, a plane channel flow is simulated. Periodic boundary conditions are applied in streamwise ( $x$ ) and spanwise ( $z$ ) directions, and in the wall normal direction ( $y$ ), a buffer region outside the wall is included, and periodicity is imposed in the buffer region as well. A Fourier cutoff filter is applied in the wall normal direction effectively filtering the wall, and a Fourier spectral method is used. A low storage second-order Runge-Kutta method is used to time discretize the nonlinear terms and the viscous terms are treated using an integrating factor.

### 3.1. THE EVOLUTION OF SMALL AMPLITUDE DISTURBANCES

In this test, evolution of a linear stability mode is illustrated. The exact solution in this case has the form:

$$u(x, y, t) = (1 - y^2) + \epsilon \operatorname{Re}\{\psi_y(y)e^{i(\alpha x - \omega t)}\}, \quad (5)$$

$$v(x, y, t) = \epsilon \operatorname{Re}\{i\alpha\psi(y)e^{i(\alpha x - \omega t)}\}, \quad (6)$$

where  $\psi$  is the Orr-Sommerfeld eigenfunction (appropriately normalized),  $\omega$  is the complex frequency (eigenvalue),  $\alpha$  is the prescribed wave number, and  $\epsilon$  is the perturbation amplitude. The simulation was performed with  $\alpha = 1$  at Reynolds numbers  $Re_c = 10000$ . For this case, the most unstable mode has eigenvalue  $\omega = 0.23752649 + i0.00373967$ . An initial condition was constructed from the eigenfunction with the most unstable mode and then filtered. Two cases were run, with 64 and 256 Fourier modes in the wall normal direction. A total of 20 "points" are in the buffer domain outside of the channel ( $|y| > 1$ ).

The exact unfiltered pressure fluctuations are formally zero in the exterior, resulting in a discontinuity in pressure, and the resulting Gibbs phenomenon in the filtered pressure is shown in figure 1c. The wall normal pressure gradient appears in the  $v$ -momentum equation, and this quantity is dominated by the filtered delta function at the boundary and the resulting Gibbs phenomenon (figure 1d). Yet the Gibbs phenomenon in velocity perturbations in figure 1a and 1b is imperceptible. The reason is that the terms  $b_v$  (figure 1e) has exactly the same structure as the pressure gradient and cancel the Gibbs phenomenon (figure 1f). The role of the boundary terms in the momentum equation is thus clear. They regularize the stress discontinuities at the wall (both pressure and viscous stresses).

In these tests, the intent is to use a filter width sufficiently fine that the only input is the filtering of the boundary (i.e. a "DNS").

The average growth rate ( $\omega_i$ , imaginary part of the complex frequency) of the disturbance was measured as a function of time. For  $\alpha = 1$ , the complex frequency ( $\omega$ ) is given by

$$\omega = -i(1/\hat{v})(d\hat{v}/dt) \quad (7)$$

Due to errors introduced in the initial conditions, there is a transient before the asymptotic growth rate is reached. The solution with 64 grids in  $y$  direction under predicts the growth rate (.00286091) and this is apparently due to inadequate resolution in the domain. The growth rate of the disturbance is correctly reproduced by the simulation with  $N_y = 256$  (.00374844 vs. the value from linear theory .00373967).

This is a severe test for the filtered boundary approach, since the Fourier spectral method behaves poorly in presence of discontinuities and because

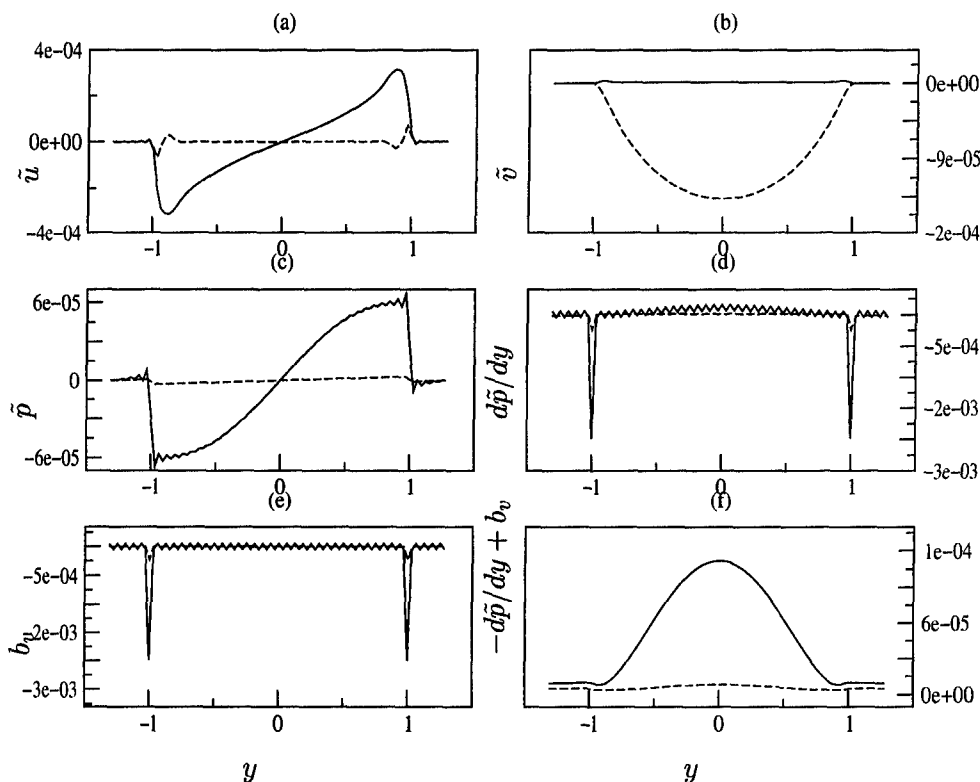


Figure 1. Effect of the boundary terms in the test of evolution of small disturbances in a channel flow. (a) filtered  $u$  velocity, (b) filtered  $v$  velocity, (c) filtered pressure, (d) pressure gradient, (e) boundary term for  $v$  equation, (f) pressure gradient + boundary term. — real part, - - - imaginary part.

the growth rate is very sensitive. But, the boundary terms precisely account for the poor behavior of the filter and provides a good representation of the filtered eigenfunction.

### 3.2. TURBULENT FULLY DEVELOPED CHANNEL FLOW

To demonstrate the applicability of this technique in simulating turbulent flow, a fully developed channel flow is computed on a  $64 \times 128 \times 64$  grid with 20 point in the buffer region. Based on the wall shear velocity  $u_\tau$  and channel half-width  $\delta$ , the Reynolds number

$$Re_\tau = \frac{u_\tau \delta}{\nu} \quad (8)$$

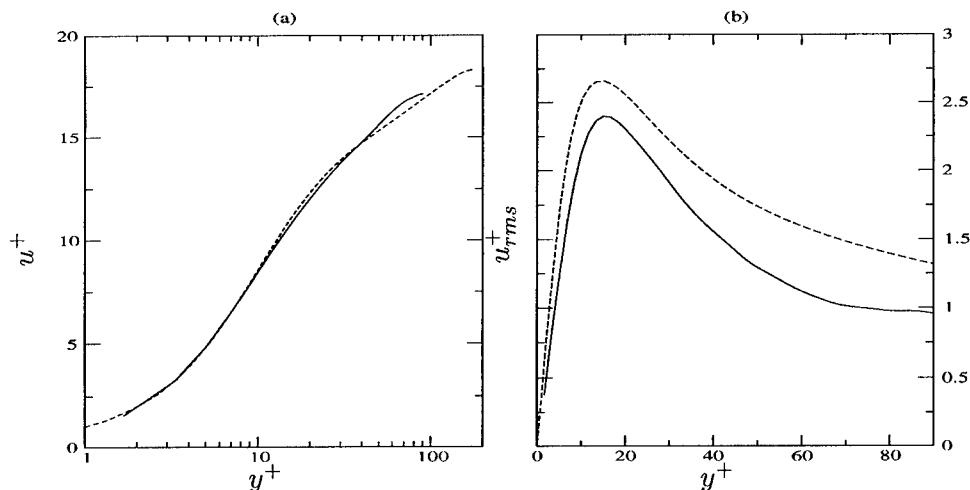


Figure 2. (a) Mean velocity, (b)  $u_{rms}$ , normalized by wall shear stress. — present simulation  $Re_\tau = 90$ , - - - Kim et. al. (1987)  $Re_\tau = 180$ .

of this flow is 92.23. The corresponding Reynolds number based on the centerline velocity and channel half-width is 1485. The streamwise and spanwise dimensions of the channel are  $4\pi\delta$  and  $4/3\pi\delta$  respectively. The flow field was initialized by spatially filtering a DNS flow field obtained at  $Re_\tau = 180$  by Kim et al. (1987).

The governing equations were integrated until the mean flow reached statistical equilibrium (approximately  $4\delta/u_\tau$ ). Instantaneous results are shown at non dimensional time 7. The mean velocity distribution normalized by the wall shear velocity is shown in figure 2a, along with the data reported by (Kim et al., 1987) at  $Re_\tau = 180$ . Also shown in figure 2b is the streamwise turbulent intensity. Note that the computed peak is 9.84% lower than the  $Re_\tau = 180$  case, and this is not a low Reynolds number effect (Keefe et al., 1992). The cause of this low peak in  $u_{rms}$  is not clear at this point, but the near wall resolution may be responsible. In other words, the turbulence near the wall is consistent with expectations for wall bounded flows. For example, familiar flow structures (streaks, inclined shear layers) are present near the wall and are approximately of the expected scale.

#### 4. Conclusion

A new approach for treating boundaries in the context of LES, especially extended boundaries, was developed, and applied to the computation of turbulent channel flow. In this representation, the issue of highly inhomogeneous filtering near a wall in LES is obviated, since homogeneous filters

can be used since the boundary itself is filtered. In this formulation, the unfiltered stresses at the wall are required, and are obtained from a minimization of perturbations in the buffer domain.

The method was successfully tested in the evolution of a Tollmien-Schlichting wave in a channel flow. Also a low Reynolds number fully developed turbulent channel was simulated. The new approach is promising for the treatment of boundaries in LES, and the application of optimal LES (Langford and Moser, 1999) to the filtered boundary formulation is planned.

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